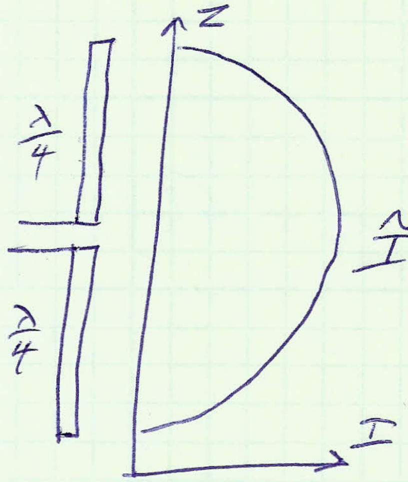
Half wave dipole antenna

from Hertzian dipole: $\tilde{E}_\theta = \frac{j I_0 l k \eta_0}{4\pi}\left(\frac{e^{-jkR}}{R}\right)\sin\theta$

$$\tilde{H}_\phi = \frac{\tilde{E}_\theta}{\eta_0}$$



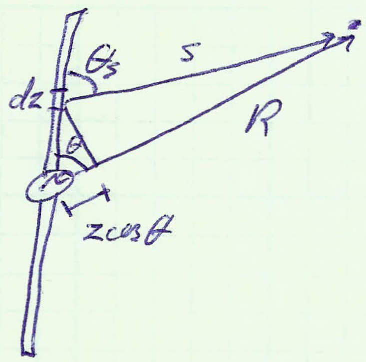
$$\tilde{I}(z) = I_0 \cos kz$$

extend expression for E, H to arbitrary current distribution

$$d\tilde{E}_\theta(z) = \frac{jk\eta_0}{4\pi}\tilde{I}(z)dz\left(\frac{e^{-jks}}{s}\right)\sin\theta$$

$$d\tilde{H}_\phi(z) = \frac{d\tilde{E}_\theta(z)}{\eta_0}$$

$$\tilde{E}_\theta = \int_{-\lambda/4}^{\lambda/4} d\tilde{E}_\theta$$



Two approximations:

① $\frac{1}{s} \approx \frac{1}{R}$ (very little effect on magnitude)

② for phase term $s \approx R - z\cos\theta$ (parallel ray approximation)

$$d\tilde{E}_\theta = \frac{jk\eta_0}{4\pi} I(z)dz\left(\frac{e^{-jkR}}{R}\right)\sin\theta\, e^{jkz\cos\theta}$$

$$\tilde{E}_\theta = \int_{-\lambda/4}^{\lambda/4} \frac{jk\eta_0}{4\pi} \overbrace{I_0 \cos kz} \left(\frac{e^{-jkR}}{R}\right)\sin\theta\, e^{jkz\cos\theta}\, dz$$

$$= j60 I_0 \left[\frac{\cos\left[\frac{\pi}{2}\cos\theta\right]}{\sin\theta}\right]\left(\frac{e^{-jkR}}{R}\right) \quad \text{using } \eta_0 \approx 120\pi$$

$$S(R,\theta) = \frac{|\tilde{E}_\theta|^2}{2\eta_0} = \underbrace{\frac{15}{\pi}\frac{I_0^2}{R^2}}_{S_0 = S_{max} = \frac{15}{\pi}\frac{I_0^2}{R^2}} \left[\overbrace{\frac{\cos^2\left[\frac{\pi}{2}\cos\theta\right]}{\sin^2\theta}}^{F(\theta)} \right]$$

pattern is similar to Hertzian dipole

Directivity of half-wave dipole:

$$P_{rad} = R^2 \iint_{4\pi} S(R,\theta)\, d\Omega$$

$$= \frac{15 I_0}{\pi} \int_0^{2\pi}\int_0^{\pi} \left[\frac{\cos\left[\frac{\pi}{2}\cos\theta\right]}{\sin\theta}\right]^2 \sin\theta \, d\theta \, d\phi$$

Must be integrated numerically

$$P_{rad} \approx 36.6 I_0^2$$

$$D = \frac{4\pi R^2 S_{max}}{P_{rad}} = \frac{4\pi R^2}{36.6 I_0^2}\left(\frac{15 I_0^2}{\pi R^2}\right) = 1.64$$

slightly higher than Hertzian dipole, with $D = 1.5$

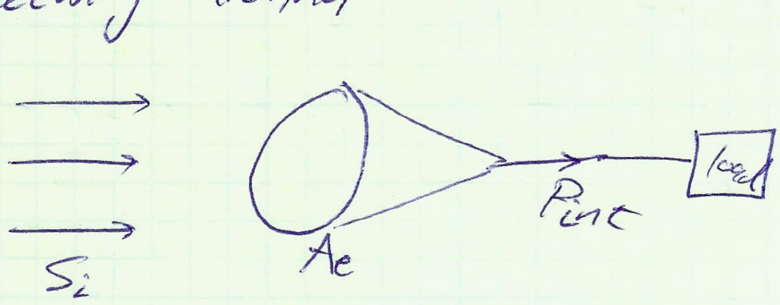
Radiation resistance:

$$R_{rad} = \frac{2 P_{rad}}{I_0^2} = \frac{2 \times 36.6 I_0^2}{I_0^2} = 73\,\Omega$$

This is orders of magnitude greater than Hertzian dipole

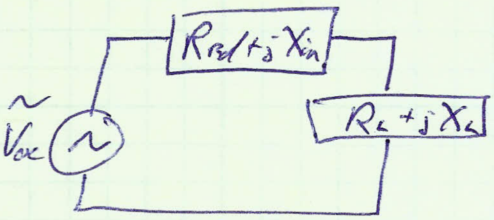
Effective area of receiving antenna

$$A_e = \frac{P_{int}}{S_i}$$



$$Z_{in} = R_{rad} + jX_{in}$$

$$Z_L = R_L + jX_L$$



for maximum power transfer, $R_{rad} = R_L$, $X_{in} = -X_L$

equivalent to $\hat{V}_{oc}$ applied to $2R_{rad}$

$$P_L = \frac{1}{2}|\tilde{I}_L|^2 R_{rad} = \frac{1}{2}\left[\frac{|\tilde{V}_{oc}|}{2R_{rad}}\right]^2 R_{rad} = \frac{|\tilde{V}_{oc}|^2}{8R_{rad}}$$

Incident power density: $S_i = \dfrac{|\tilde{E}_i|^2}{2\eta_0} = \dfrac{|\tilde{E}_i|^2}{240\pi}$

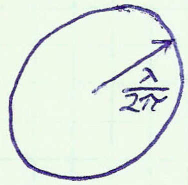
$$A_e = \frac{P_{int}}{S_i} = \frac{30\pi|\tilde{V}_{oc}|^2}{R_{rad}|\tilde{E}_i|^2}$$

For the Hertzian dipole, $\tilde{V}_{oc} = \tilde{E}_i l$, and $R_{rad} = 80\pi^2\left(\frac{l}{\lambda}\right)^2$

$A_e = \dfrac{3\lambda^2}{8\pi}$ remember that $D = \frac{3}{2}$ for Hertzian dipole

In general $A_e = \dfrac{\lambda^2 D}{4\pi}$

For isotropic antenna, $A_e = \dfrac{\lambda^2}{4\pi}$



Friis Transmission Formula

$$S_{iso} = \frac{P_t}{4\pi R^2}$$

$$S_r = G_t S_{iso} = \frac{G_t P_t}{4\pi R^2} \qquad (G_t = \varepsilon_t D_t)$$

$$P_r = S_r \cdot A_r \cdot \varepsilon_r$$

$$= \frac{G_t P_t}{4\pi R^2} \cdot \underbrace{\frac{\lambda^2 D_r}{4\pi} \cdot \varepsilon_r}_{G_r}$$

$$\frac{P_r}{P_t} = G_t G_r \left(\frac{\lambda}{4\pi R}\right)^2$$

Example: 100 kW radio station 10 km away, broadcasting at 100 MHz

assume $G_t \approx G_r \approx 1$

$\lambda = 3\,\text{m}$ at 100 MHz

$\left(\frac{3}{4\pi \times 10000}\right)^2 = 5.7\times10^{-10}$, or 92 dB attenuation

$P_r = 0.000057$ watts